

MAS212 Linear Algebra I 2007-2008

Coursework 5

Please hand in your solutions of the **starred** exercises by **2pm on Friday 2 November** using the [blue Linear Algebra I Collection Box](#) on the Ground Floor.

Don't forget to put your **name** and **student number** on your solutions, and to **staple** them.

Exercise* 1. (2 marks) Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in a vector space. Prove that if $\mathbf{u} + \mathbf{v} = \mathbf{w} + \mathbf{v}$ then $\mathbf{u} = \mathbf{w}$.

Exercise* 2. (4 marks) In each case, determine whether S is a subspace of $\mathbb{R}^3$ and give reasons for your answer:

(a) $S = \{ (1, s, t)^T \mid s, t \in \mathbb{R} \},$

(b) $S = \{ (r, s, t)^T \mid r, s, t \in \mathbb{R} \text{ and } r + 2s - t = 0 \},$

(c) $S = \{ (r, s, t)^T \mid r, s, t \in \mathbb{R} \text{ and } s^2 + t^2 \leq 1 \}.$

Exercise* 3. (6 marks) Which of the following statements (if any) are true? Justify your answers.

(a) $\{ A \in \mathbb{R}^{n \times n} \mid A \text{ is symmetric} \}$ is a subspace of $\mathbb{R}^{n \times n}.$

(b) $\{ A \in \mathbb{R}^{2 \times 2} \mid A \text{ is singular} \}$ is a subspace of $\mathbb{R}^{2 \times 2}.$

(c) $\{ \mathbf{f} \in C[-1, 1] \mid \mathbf{f}(-1) + \mathbf{f}(1) = 0 \}$ is a subspace of $C[-1, 1].$

(d) $\{ \mathbf{p} \in P_7 \mid \mathbf{p} \text{ has degree greater than } 4 \}$ is a subspace of $P_7.$

Exercise* 4. (3 marks) Determine the nullspaces of the following matrices

$$A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & -3 & -2 \\ 2 & 4 & -5 & 1 \\ -3 & -6 & 8 & 1 \end{pmatrix}.$$

Exercise 5. Let F be a fixed 3×2 matrix, and define $S = \{ A \in \mathbb{R}^{2 \times 4} \mid FA = O_{3 \times 4} \}$. Is S a subspace of $\mathbb{R}^{2 \times 4}$?

Exercise 6. Let U and V be subspaces of a vector space W . Define their **intersection** $U \cap V$ and their **sum** $U + V$ by

$$U \cap V = \{ \mathbf{w} \in W \mid \mathbf{w} \text{ belongs to both } U \text{ and } V \},$$

$$U + V = \{ \mathbf{w} \in W \mid \mathbf{w} = \mathbf{u} + \mathbf{v} \text{ where } \mathbf{u} \in U \text{ and } \mathbf{v} \in V \}.$$

Show that $U \cap V$ and $U + V$ are subspaces of W .

Exercise* 7. (5 marks) Which of the following statements (if any) are true? Justify your answers.

(a) $\{ (1, 0, 0)^T, (3, 1, 1)^T, (0, 1, 2)^T \}$ is a spanning set for $\mathbb{R}^3.$

(b) $\{ (1, 0, 0)^T, (3, 1, 1)^T, (0, 1, 2)^T, (-1, 3, 4)^T \}$ is a spanning set for $\mathbb{R}^3.$

(c) $\{ E_{11}, E_{12}, E_{21}, E_{22} \}$ is a spanning set for $\mathbb{R}^{2 \times 2}$, where

$$E_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad E_{12} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad E_{21} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad E_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

(d) $\{ \mathbf{p}_1, \mathbf{p}_2 \}$ is a spanning set for P_2 , where $\mathbf{p}_1(x) = 2 + x$, $\mathbf{p}_2(x) = x^2$.